MODEL INDEPENDENT EXTRACTIONS OF $|V_{ub}|$ FROM INCLUSIVE B DECAYS

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We discuss the possibility of extracting $|V_{ub}|$ from various spectra of inclusive B decays, without large systematic errors which usually arise from having to model the Fermi motion of the heavy quark. This strategy can be applied to the electron energy spectrum, as well as the hadronic mass spectrum. Modulo violation of local hadron-parton duality, the theoretical error in the extraction is estimated to be less than 10%.

Among the nine entries of the Cabbibo-Kobayashi-Maskawa (CKM) matrix, V_{ub} and V_{td} are the most important elements for understanding CP violation. Unfortunately, these two elements are the most difficult to extract experimentally. When making theoretical predictions, one is often forced to resort to various models due to our inability to calculate hadronic dynamics from QCD. The trouble with models is that we have no idea what the theoretical uncertainty is. In what follows I will describe a method of extracting $|V_{ub}|$ from the inclusive semileptonic decays $B \to X_u \ell \nu$ without having to model hadronic dynamics.

In the standard model the decay of B mesons can be calculated in a systematic expansion in α_s, $1/(M_W, m_t)$ and $1/m_b$. First the full standard model is matched onto the four fermion theory. It is then run down to the scale m_b, after which the decay rate may be calculated in heavy quark effective theory (HQET) in expansion in $1/m_b$. Using the optical theorem, the inclusive decay rate can be computed by taking the imaginary part of the forward scattering amplitude and integrating along the physical cut in the complex $v \cdot q$ plane, where q is the momentum transfered to the lepton pair and v is the four velocity of the B meson. The integration contour can then be deformed away from the resonance region into a region where perturbative calculation can be trusted.[1] Therefore, a hadronic quantity averaged over a sufficient number of states can be computed reliably using QCD perturbation theory. This is the justification of (global) parton-hadron duality.

Unfortunately, at the present stage we are unable to measure the total decay rate of $b \to u \ell \nu$ due to the overwhelming charmed background. It is thus necessary to introduce experimental cuts on various differential spectra. The traditional method is to put a cut on the lepton energy, $M_B/2 > E_\ell > (M_B^2 - M_D^2)/(2M_B)$, which leaves us with a window of $2\Delta E/m_b \approx 0.13$. It is also possible to use the cut hadronic invariant mass spectrum[2] by considering hadronic final states with invariant mass $M_D^2 > s_H > 0$, leaving a window of $\Delta s_H/m_b^2 \approx 0.15$. Recently it has been proposed that one can also use the cut leptonic invariant mass spectrum[3] $M_D^2 > q^2 > (M_B - M_D)^2$, with a window of $\Delta q^2/m_b^2 \approx 0.57$. In any case, the necessity of experimental cut introduces a third scale ρ, in addition to m_b and $\Lambda_{\rm QCD}$, which measures the relative size of the phase space of interest.

To see the effect of ρ on the theoretical calculation, let us consider the end-point region of the lepton energy spectrum. At $\mathcal{O}(\alpha_s)$ the spectrum contains infrared and collinear logarithms like $\log^2(1-x)$ and $\log(1-x)$, where $x = 2E_\ell/m_b$. In the end-point region, $\alpha_s(m_b)\log^2(1-\rho) \approx 0.8$, which signifies the poor convergence of QCD perturbation theory. One therefore needs to resum logarithms of the form $\alpha_s^m \log^n(1-x)$. As for the non-perturbative expansion in $1/m_b$, it has singular terms[4] $\delta(1-x)$ and $\delta'(1-x)$. This suggests that the relevant expansion parameter in this region is $\Lambda/[m_b(1-\rho)] \sim \mathcal{O}(1)$ and one needs to take into account an infinite number of terms like $\delta^{(n)}(1-x)$ all contributing at leading order.

To achieve the above goals, it is useful to utilize the infrared factorization in the lepton energy end-point spectrum.[5] It can be shown that, when $x \to 1$, $s_H \sim \mathcal{O}(m_b(1-x))$ and $E_H \sim \mathcal{O}(m_b)$. The u-quark produces a *jet* of collinear particles whose invariant mass approaches zero with total energy held fixed. In addition, the jet hadronizes at a much later time in the rest frame of the B meson, due to the time dilation. The differential decay rate thus factorizes into three parts, the hard, the jet, and the soft, characterized by three disparate scales: $\mathcal{O}(m_b)$, $\mathcal{O}(m_b\sqrt{1-x})$, and $\mathcal{O}(m_b(1-x))$ respectively,

$$\left.\frac{d^3\Gamma}{dx dq^2 d(v\cdot q)}\right|_{x\to 1} \sim \int dz\ S(z,\mu)J(z,\mu)H(\mu), \tag{1}$$

where μ is the factorization scale. Physical amplitudes should not depend on μ and this leads to a renormalization group equation which can be used to resum the infrared and collinear logs.[5,6] Taking the moment of the spectrum further leads to

$$\begin{aligned} M_N &= -\frac{1}{\Gamma_0}\int_0^{M_B/m_b} dx\ x^{N-1}\frac{d}{dx}\frac{d\Gamma}{dx} \\ &= f_N \sigma_N J_N H_N + \mathcal{O}(1/N) \end{aligned} \tag{2}$$

H_N contains the short distance interaction at scale m_b and can be computed perturbatively. σ_N and J_N contain the infrared and collinear logs, which are to be resummed using RG equation. f_N contains interaction at scale $\Lambda_{\rm QCD}$ and is strictly non-perturbative. A formal expression for f_N in the x-space is

$$f(k_+) = \langle B(v)|\ \bar{b}_v\ \delta(k_+ - iD_+)\ b_v\ |B(v)\rangle, \tag{3}$$

which is the light-cone distribution function of the b-quark inside the B meson and resums the most singular terms like $\delta^{(n)}(1-x)$ in HQET.[7] Since $f(k_+)$ contains the long distance physics and is not calculable from QCD, one typically has to resort to modeling. On the other hand, this implies that $f(k_+)$ is insensitive to short distance physics and thus universal in inclusive B decays. One can extract $f(k_+)$

from the inclusive $B \to X_s\gamma$ decays. In the end-point region of photon spectrum similar infrared factorization holds and the moment factorizes into[5,8]

$$\begin{aligned} M_N^\gamma &= -\frac{1}{\Gamma_0^\gamma}\int_0^{M_B/m_b} dx^\gamma (x^\gamma)^{N-1}\frac{d\Gamma^\gamma}{dx^\gamma} \\ &= f_N\sigma_N J_N^\gamma H_N^\gamma \end{aligned} \tag{4}$$

Replace $f_N\sigma_N$ in (2) by M_N^γ using (4) and take the inverse Mellin transform to go back to x-space we obtain[9]

$$\frac{|V_{ub}|^2}{|V_{ts}^*V_{tb}|^2} \sim \frac{\delta\Gamma(B \to X_u\ell\nu,\rho)}{\int\int_\rho \frac{d\Gamma^\gamma}{dx^\gamma} * K(x^\gamma;\alpha_s)}, \tag{5}$$

where the kernel $K(x^\gamma;\alpha_s)$ can be computed perturbatively. Corrections to this are $\mathcal{O}(\Lambda/m_b, \alpha_s(1-\rho), (1-\rho)^3)$. These corrections are of order 10% in $|V_{ub}|^2$.

However, this assumes that parton-hadron duality works well. In fact, the end-point region of the electron energy spectrum contains only about 10% of the total rate, whereas the low s_H region of the hadronic invariant mass spectrum contains 40-80% of the total rate. One therefore expects that parton-hadron duality works better in this case. A similar strategy to extract $|V_{ub}|$ can be applied.[10] The leptonic invariant mass spectrum might be a theoretically clean method[3], but the cut contains only 20% of the total rate. In the end it is clear that no single extraction of $|V_{ub}|$ from inclusive decays should be trusted. We would have faith only after convergence among several independent extractions. Comparision among different extractions would also shed some light on the validity of parton-hadron duality.

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